ſort of phænomenon, never hitherto honoured with an adequate deſcription. If it ſhould appear to the Royal Society in the ſame light, they will excuſe the trouble given on this occaſion by,

SIR,

Your much obliged,

and moſt obedient,

humble ſervant,

Chriſt-Church, Oxon.
Aug. 29, 1764.

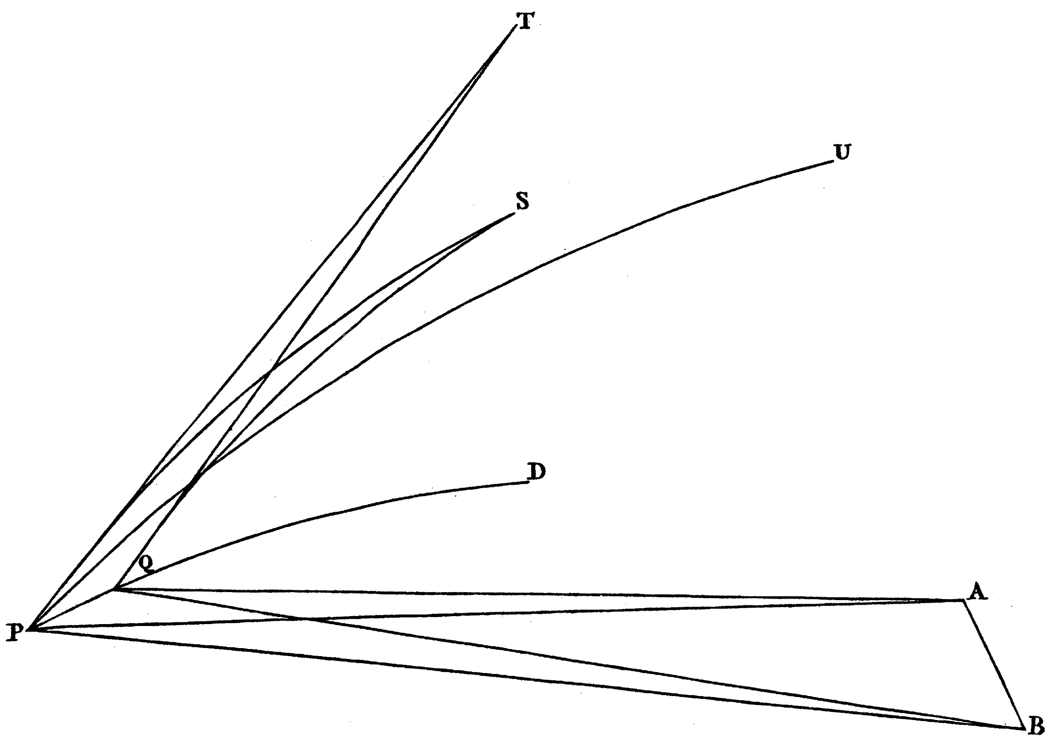
John Swinton.

LVI. *Some Remarks upon the Equation of Time, and the true Manner of computing it. By* Nevil Maſkelyne, *A. M. Fellow of* Trinity College, Cambridge, *and F. R. S.*

Read Dec. 13, 1764.

THESE remarks were wrote above a twelve-month ago, and would have been then communicated to the Royal Society, had not my voyage to Barbados prevented it. Since my return from thence, I find part of the miſtakes here pointed out acknowledged and corrected by M. Delalande, in his Treatiſe of Aſtronomy lately publiſhed, to whom I remember to have communicated my ideas on the ſubject, when he was in England. Nevertheleſs, as the error ariſing from taking

Philos. Trans. Vol. LIV. TAB. XX. p. 337.



J. Mynde sc.

taking the equation of the equinoctial points into the account ſtill remains uncorrected by him; and as I flatter myſelf, that what is ſaid here may tend to ſet the whole matter in a clear light, I apprehend the publication of theſe remarks may ſtill be proper.

The French Almanack, called the Connoiſſance des Mouvements Celeſtes, hath been deſervedly eſteemed by aſtronomers, as the moſt complete publication of its kind. Its preſent learned editor M. Delalande hath rendered its uſe more extenſive by making the calculations from the lateſt and moſt approved tables, and alſo adding ſuch explanations of them, as, at the ſame time, lay open before his readers the moſt conſiderable improvements of modern aſtronomy. Nevertheleſs, as the beſt mathematicians are not infallible, ſo I have reaſon to think I have diſcovered ſome errors in M. Delalande's method of computing the equation of time in this ephemeris, or, which comes to the ſame thing, the mean time, at the inſtant of apparent noon.

M. Delalande ſays, page of the Connoiſſance for 1760, which he repeats in the publications of other years, that, "to calculate exactly the difference between "mean and true time (that is to ſay the equation of "time) at the inſtant of apparent noon, the ſum of "the equation of the ſun's centre, the difference "between his longitude and right aſcenſion, the lunar "equation, the equations of Jupiter and Venus, and "that of the preceſſon of the equinoxes, with their "proper ſigns, muſt be converted into mean ſolar time. "He adds, that it was impoſſible, before this time, "to obtain the equation of time exactly; 1ſt, becauſe "hitherto no account has been made of the four

" little equations, the ſum of which may produce " above three ſeconds of time; 2dly, becauſe it has " been the practice to convert the equation of the " ſun's centre, and the difference between his right " aſcenſion and longitude into time of the Primum " Mobile, inſtead of converting them into mean " ſolar time, which, ſays he, may produce an error " of two ſeconds and a half; 3dly, becauſe the " equation of the ſun's centre was not known ex- " actly before, every minute of which anſwers to " four ſeconds in the equation of time."

I readily agree with M. Delalande, that the equation of time could not be had ſo exactly formerly, as it may now, when we have a much more exact theory of the ſun, and are lately made acquainted with new equations of his motion. I cannot, however, aſſent to his poſition, that the equation of the equinoctial points is to be taken into this account, together with the other equations, ſince this is not an inequality in the ſun's motion, but ariſes from a motion of the equator itſelf; yet of ſuch a kind as cannot accelerate or retard the coming of the ſun, or any ſtar lying within the tropics, to the meridian, by above a quarter of a ſecond of time. This will, perhaps, appear in a good meaſure plain, if it be conſidered, that the diurnal motion of the earth round its axis is neither accelerated nor retarded by the action of the ſun and moon in producing the preceſſion of the equinoxes, and variations of the inclination of the earth's axis to the ecliptic. The effect of theſe actions is, that the terreſtrial pole, each day, deſcribes a ſmall arc of a circle about the centre of the earth, in the plane of a celeſtial meridian paſſing through

through the ſun or moon, or rather one between both; and, conſquently, the equator of the earth has its motion in its own plane neither accelerated nor retarded, but obtains a new motion, wnoſe axis is one of its own diameters. This is the true origin, as well of the minuter and periodical nutations, as of the regular and perpetual motion of the earth's axis about the pole of the ecliptic, obſerved in all ages, on which the continual preceſſion of the equinoxes depends.

But, to illuſtrate more fully the point in queſtion, let P, ſee fig.   repreſent the north pole of the celeſtial equator, which ſuppoſe to be tranſlated, in any certain time, from P to Q, through the ſmall ſpace P Q, upon the meridian P D, by the actions of the ſun and moon; let A be the equinoctial point of aries, and S the ſun or ſtar. It is evident, that, as the rotation of the earth round its axis is no way affected, the tranſlation of the celeſtial pole from P to Q along the arch P Q, of the celeſtial meridian P D, will occaſion no alteration in the time of any given meridian of the earth coming to the fixt celeſtial meridian P D, nor conſequently in the time of the ſun or ſtars, when lying in this meridian, appearing to paſs the meridian of the given place; contrary to what ſhould follow from the method of computing the equation of time, uſed in the Connoiſſance des Mouvements Celeſtes; according to which, as long as the equation of the equinoxes is any thing, the equation of time muſt be affected thereby, and conſequently the abſolute time of the ſun's paſſing the meridian.

But,

But, if the ſun or ſtar lie not in the celeſtial meridian P D, but in ſome other meridian P S, at S, then the ſpherical angle S P D is the diſtance of the ſun from the meridian P D, when the pole is at P, and S Q D is his diſtance from the ſame meridian, when the pole is tranſlated to Q. Let P T, Q T, meeting in T, be tangents of the meridians P S, Q S, in P and Q; T Q D being the external angle of the rectilineal triangle T P Q, the angle P T Q is = T Q D − T P D = S Q D − S P D, and, therefore, is a meaſure of the alteration of the time of any meridian of the earth's coming to the ſun at S, produced by the tranſlation of the pole from P to Q. Now the ſine of P T Q is to the ſine of T P Q, as P Q to T Q; whence, calling the radius unity, and taking P Q, on account of its ſmallneſs, = the ſine of P Q, and the angle P T Q = the ſine of P T Q, we have $P\,T\,Q = \frac{P\,Q \times \text{ſine } T\,P\,Q}{T\,Q}$ = the tranſlation of the pole × the ſine of the right aſcenſion of the ſun or ſtar reckoned from the meridian in which the pole moves, ÷ the tangent of the polar diſtance, or, which is the ſame thing, × the tangent of the declination. Therefore, as P Q, ariſing from the nutation of the earth's axis, never exceeds $9''\frac{1}{2}$, the greateſt value of P T Q, for the ſun can never exceed $9''\frac{1}{2} \times$ tangent of $23^\circ\frac{1}{2}$ the ſun's greateſt declination, $= 4'',1$ which anſwers to about $\frac{1}{4}$ of a ſecond of time: and ſo much, and no more, may the ſun come ſooner or later to the meridian, on account of the nutation of the earth's axis: whereas, if the equation of the equinoxes was to be applied directly in the computation, according to M. Delalande's

M. Delalande's method, it would ſometimes, namely when at its maximum of 18″, produce nearly 1 $\frac{1}{4}$ ſecond of time.

But, tho' this demonſtration may be admitted to be juſt, yet it may perhaps be aſked, wherein lay the fault of the method of computation here cenſured, and whether the time of the ſun's coming to the meridian is not regulated by his right aſcenſion? It may alſo be thought requiſite, that the true manner of computing the equation of time, from the ſun's right aſcenſion, ſhould be ſhewn.

Firſt, let it be obſerved, that when the pole is at P, A is the equinoctial point, and, when the pole is tranſlated to Q, ſome other point B is the equinoctial point: therefore the ſun's mean right aſcenſion U P A is reckoned from A, and his apparent right aſcenſion B Q S, computed from his longitude, corrected by the equation of the equinoxes A B, or B S, is reckoned from another point B. Now the equation of time is proportional to the difference between the ſun's mean and true right aſcenſion, both reckoned from the ſame point; ſo that if the ſun's mean right aſcenſion is reckoned from A, his apparent right aſcenſion, in this caſe, ſhould be reckoned from A too; or if the apparent right aſcenſion is reckoned, more properly, from the apparent right equinox B, his mean right aſcenſion, for this purpoſe, ſhould be reckoned from B likewiſe. For it is plain, from what has been ſaid above, that no ſmall motion of the pole P can at all affect the abſolute time of a ſtar in the equator's coming to the meridian of any place; for, the tangent Q T then becoming infinite, the angle P T Q vaniſhes; therefore the mean equi-

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nox A will come to the meridian at the ſame inſtant of abſolute time, as if the pole had not been tranſlated from P to Q; and the difference of time between the ſun S coming to the meridian, and a fiictitious ſun U, ſuppoſed to move uniformly in the equator, with a motion equal to the ſun's mean motion in longitude, or the equation of time will be therefore meaſured by A Q S ∽ A P U, the difference of their right aſcenſions reckoned from the ſame point A. It will alſo, by the like reaſoning, be meaſured by B Q S ∽ B P U, the difference of their right aſcenſions reckoned from the ſame point B; for B being the equinox, when the pole is at Q, the abſolute time of the point B paſſing the meridian of any place will remain the ſame as if the pole had continued at P; whence the propoſition eaſily follows, in like manner as above.

It may be now proper to ſhew how the equation of time, as affected by the nutation of the earth's axis, ought to be computed. This may be done two ways. The " firſt follows from what has been juſt " laid down: correct the mean right aſcenſion of " the ſun U P A, by the preceſſion of the equinoxes " in right aſcenſion A P B (which is always to the " preceſſion in longitude B A, as coſine of the obli- " quity of the ecliptic, to the radius, or as 12 to " 13 nearly) the difference of the ſun's mean right " aſcenſion thus corrected B P U, and the ſun's appa- " rent aſcenſion B Q S, turned into time, is the true " equation of time."

Otherwiſe the effect of the nutation of the earth's axis upon the equation of time, if thought deſerving notice, as it can never exceed $\frac{1}{4}$ of a ſecond of time,

time, might be computed from the angle $PTQ = PQ \times$ fine of $TPD \div TQ$, which, fuppofing the nutation of the pole to be performed in a circle, whofe radius is 8″, or a mean between the two conjugate femi-axes of the ellipfis, in which it really moves, is $= 8'' \times$ tangent of the fun's declination $\times$ cofine of the difference of fun's right afcenfion, and the longitude of the moon's afcending node.

But this is not the only miftake in the computation of the equation of time in the Connoiffance des Mouvements Celeftes, though it may exceed one fecond of time. M. Delalande fays that the fum of the equation of the fun's centre, the difference between his longitude and right afcenfion, and the fum of the four little equations, muft be converted into mean folar time, in order to find the equation of time; and adds, that no exact equation table could be had, before this time, for three reafons, one of which is, that it has always been the practice to convert the equation of the fun's centre and the difference between his longitude and right afcenfion into time of the Primum Mobile, inftead of mean folar time, which, fays he, may produce an error of $2\frac{1}{2}$ feconds.

Now I muft here freely own, that as I could not, without fome reluctance, and only from the fulleft proof, allow all the mathematicians and aftronomers, before this time, to have been miftaken in the manner of converting the quantities above-mentioned into time, fo I can find no reafon to conclude fo from what has been cited above: on the contrary, from a full confideration of the fubject, I apprehend the method hitherto ufed by the mathematicians to

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be juſt, and that the author has himſelf fallen into an equal miſtake with that of which he accuſes them.

But, in order to ſet this matter in a clearer light, it will be firſt neceſſary to conſider motion and time, relatively to each other; for, except this be done, it will be impoſſible to underſtand any thing preciſe from converting a certain number of minutes and ſeconds into mean ſolar time, or time of the Primum Mobile.

There are three different kinds of time uſed by aſtronomers, ſidereal time, apparent ſolar time, and mean ſolar time. The interval between the tranſit of the firſt of Aries acroſs the meridian one day, and its return to it the next day, is called a ſidereal day, which is divided into 24 equal parts or hours, and the hours into minutes, *&c.* This time is ſhewn by a clock regulated to agree with the tranſit of the ſtars acroſs the meridian. The interval between the tranſit of the ſun acroſs the meridian one day, and his tranſit the next day, is called an apparent ſolar day, which is divided into hours, minutes, *&c.* of apparent time. The ſolar day, it is manifeſt, and its hours, minutes, *&c.* are of different lengths, at different times of the year: on account of which inequality, a good clock, which keeps equal time, cannot long agree with the ſun's motion, which is unequal. Therefore, aſtronomers have deviſed an imaginary time, called mean ſolar time; which is what would be pointed out by the ſun, if his motion in right aſcenſion from day to day was uniform, or, in other words, it is what would be pointed out by a fictitious ſun or planet ſuppoſed to move uniformly in the equator, with a motion equal to the mean

motion of the ſun in longitude, its diſtance from the firſt point of Aries (meaning hereby the mean equinox) being always equal to the mean longitude of the ſun: and as apparent noon is the inſtant of the true ſun's coming to the meridian, ſo mean noon is the inſtant at which this fictitious planet would come to the meridian. The interval between its coming to the meridian on any two ſucceſſive days is a mean ſolar day, which is divided into hours, minutes, *&c.* of mean ſolar time; all which it is manifeſt will preſerve the ſame length at all times of the year.

The equation of time, at the inſtant of apparent noon, or of the ſun's paſſing the meridian, being equal to the difference between mean time and 12 hours, is alſo equal to the interval between the mean and true ſun's paſſing the meridian expreſſed in mean ſolar time: to find which, we have the diſtance of the mean ſun from the meridian, at the inſtant of apparent noon, equal to the difference between the ſun's apparent and mean right aſcenſion (both reckoned either from the mean or apparent equinox) which may be called the equation of right aſcenſion. The queſtion, therefore, comes to this, How many minutes and ſeconds of mean ſolar time doth the mean ſun take to move this diſtance up to or from the meridian? Aſtronomers hitherto have allowed 1 minute of time to every 15 minutes of right aſcenſion, and ſo in proportion; and, I apprehend, juſtly too; for does not the mean ſun, in returning to the meridian, deſcribe 360° about the pole in 24 hours of mean ſolar time; whence it is plain, that his departure from the meridian is at the rate of 15° to 1 hour, and 15′ to 1 minute of mean ſolar time.

 Therefore

Therefore aſtronomers have not converted the equation of right aſcenſion into time according to the motion of the Primum Mobile; for, the equation of time being mean ſolar time, and the motion of the Primum Mobile being compleated in 23 H. 56 M. 4 S. of mean ſolar time, therefore 15° motion of the Primum Mobile does not anſwer to 1 hour of mean ſolar time (though it does to 1 hour of ſidereal time) but to the 24th part of 23 H. 56 M. 4 S. or 59 M. 50 $\frac{1}{6}$ S. And it appears, that the equation of time in the Connoiſſance des Mouvements Celeſtes has been computed in this manner, and the table in the 79th page of the Connoiſſance for 1761 has been made uſe of, entitled, "A table to convert into degrees the time of a clock regulated according to the mean motion of the ſun." The degrees of this table are evidently degrees of the Primum Mobile, 1 hour of mean ſolar time giving 15° 2′ 27,8″, which anſwers to the motion of the ſtars from the meridian, but not to the mean motion of the ſun from thence, which is 15° to 1 hour of mean ſolar time: whence it appears, that this writer hath evidently fell into the miſtake of taking motion or ſpace of the Primum Mobile, inſtead of the mean motion of the ſun from the meridian; an equal miſtake to that of which he erroneouſly ſuppoſes former mathematicians to have been guilty, in computing the equation of time. So that the equation of time in this ephemeris, beſides the miſtake ariſing from the taking in the equation of the equinoctial points into the account, is conſtantly too ſmall in the proportion of 24 hours to 23 H. 56 M. 4 S. or of 366 to 365, or too ſmall by 1 ſecond upon every 6 minutes of the equation

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equation of time: and the miſtake of 2 $\frac{1}{2}$ ſeconds, which was ſuppoſed to be found in the old manner of reducing the equation of right aſcenſion into time, really takes place in this new method; which, added to 1 ſecond of time, ariſing from the miſtake in taking the preceſſion of the equinoxes into the account, produces 3 $\frac{1}{2}$ ſeconds, an error which, I apprehend, the aſtronomical equation tables uſed ſince Mr. Flamſtead's time have but rarely exceeded.

To ſome, who are not well acquainted with the preſent improved ſtate of aſtronomy, the difference in queſtion may ſeem a matter of indifference, and too trifling for notice. But, if truth is the object of all our enquiries, why ſhould we wilfully go beſide it in the ſmalleſt matters? And is it not a juſtice due to paſt aſtronomers, to whom we owe the foundations of all our knowledge, to vindicate them even from the ſmalleſt cenſure, which they do not appear to deſerve? At the ſame time, I flatter myſelf, that the learned editor of the Connoiſſance des Mouvements Celeſtes, and alſo the friends of the late illuſtrious Abbé de la Caille, who, I believe, was inadvertently the firſt author of this miſtake, will take no offence at my endeavouring to clear up a point, which they, doubtleſs for want of having given ſufficient attention to, ſeem to have miſtaken: ſince, truth being the common object of all our purſuits, we ought candidly to accept as well the aſſiſtance we receive from each other for bringing us into the right road, when we happen to have ſtrayed from it, as for helping us forward on our journey.

The Figure referred to in p. 339. *ſhould be* TAB. XX.

 LVII. *Aſtro-*